Organizing Just Intonation Pitches through Xenakis' Sieves and Prime Decomposition

Conference Paper · May 2022

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Organizing Just Intonation Pitches through Xenakis' Sieves and Prime Decomposition

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Proceedings of the *Xenakis 22: Centenary International Symposium*Athens & Nafplio (Greece), 24-29 May 2022 - https://xenakis2022.uoa.gr/

Abstract

In this article, we will present how we can create, organize and understand pitches of Just Intonation using 1) the theory of sieves (XENAKIS, 1992) to filter the numbers of the harmonic series, 2) the decomposition by prime-numbers (EXARCHOS et al., 2011) applied to the numbers resulted from the sieve, 3) the concept of harmonic identities by Johnston (2006), and finally, 4) the Combinations-Product Set (CPS) by Erv Wilson (NARUSHIMA, 2018), that allows us to see the relationships between harmonic identities. When we understand that only prime numbers generate new harmonic identities in the interval sense (JOHNSTON, 2006), and odd-numbers create just a new pitch-class from staking prime numbers, we can apply the prime-decomposition for all the numbers of one sieve and claim that: all the non-prime numbers can be understood how intersections between distinct harmonics series. Based on Exarchos' (2011) analysis of sieves, this idea can help us find a way to use the sieves of Xenakis in the Just Intonation context. Complementary with this compositional idea, we will present two OpenMusic libraries developed to help the compositional process.

1. Introduction

The Just Intonation is a system where all the pitches can be represented using ratios, which means that all pitches have integer relations with one fundamental pitch. The theories of Harry Partch, Ben Johnston, and Erv Wilson are some efforts to develop ways to organize the infinity of possible ratios used in a compositional context.

In Partch's theory, understanding two terms are fundamental to comprehending his work: the concept of *limit* and *identity*. The former idea defines the biggest odd number used in the *Diamond Theory* (see Partch, 1974), and the term *identity* defines what Partch understands how harmonic identity. For him, all odd numbers are new harmonic identities. However, unlike Partch, we understand the idea of *identity* using Johnston's (2006, p. 27) claim: "Each *prime number* used in deriving a harmonic scale contributes to a characteristic psychoacoustical meaning." This choice will become clear ahead.

Less systematically than Partch, one of Johnston's few systems exhaustively described by himself is used in the String Quartet no. 2 and no. 3. This system proposes stacking (up and down) the intervals 4/3, 3/2, 6/5, 5/4 until the pitches overlap. At the end of the process, it creates a 53-pitch set. It is implemented in the OM patch below (note that Johnston does not define when the notes start to overlap, so this patch is an excellent approximation but not exact).

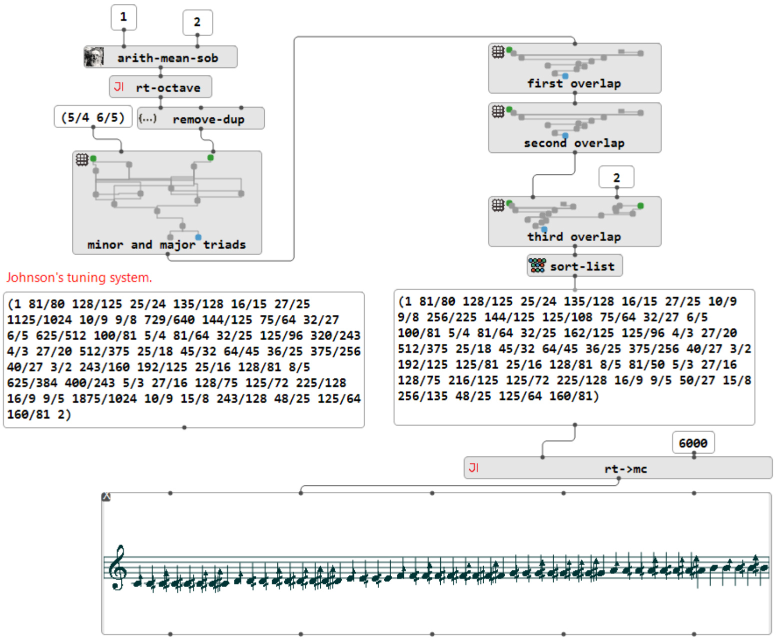

Figure 1 - OM patch to reproduce the microtonal system of String Quartet no. 2 and no. 3 of Ben Johnston.

We realize that one interesting way to see the concept of *identity* used in Partch and Johnston (2006, p. 27) is through Wilson's theory of Combinations Product Sets (CPS). See the structure named Hexany:

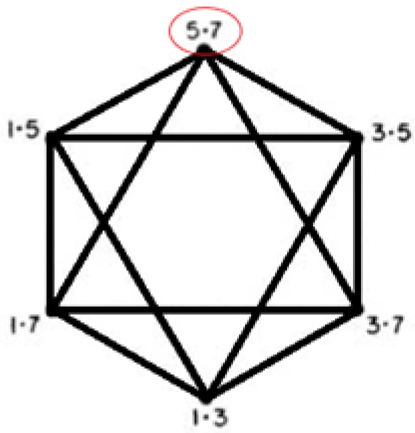

Figure 2: Hexany by Erv Wilson (Narushima, 2017, p. 153).

The main characteristic of CPS is the connection between vertices of the geometric structure. For example, the vertice (5-7) is connected with (3-5) because they share the number 5. In a musical sense, it means that (5-7) is the 7º harmonic of the number 5, and the vertice (3-5) is the 3º harmonic of the number 5 – note that the number in common in these two vertices transforms in the fundamental of one harmonic series.

Assuming the interpretation of the list of prime numbers that compose some non-prime numbers – like (3 5 7) in the prime-decomposition of 105 – how vertices in CPS's, it will be possible to see the same harmonic connections present in the Hexany (This is why we use the identity conception of Johnston). See the image below:

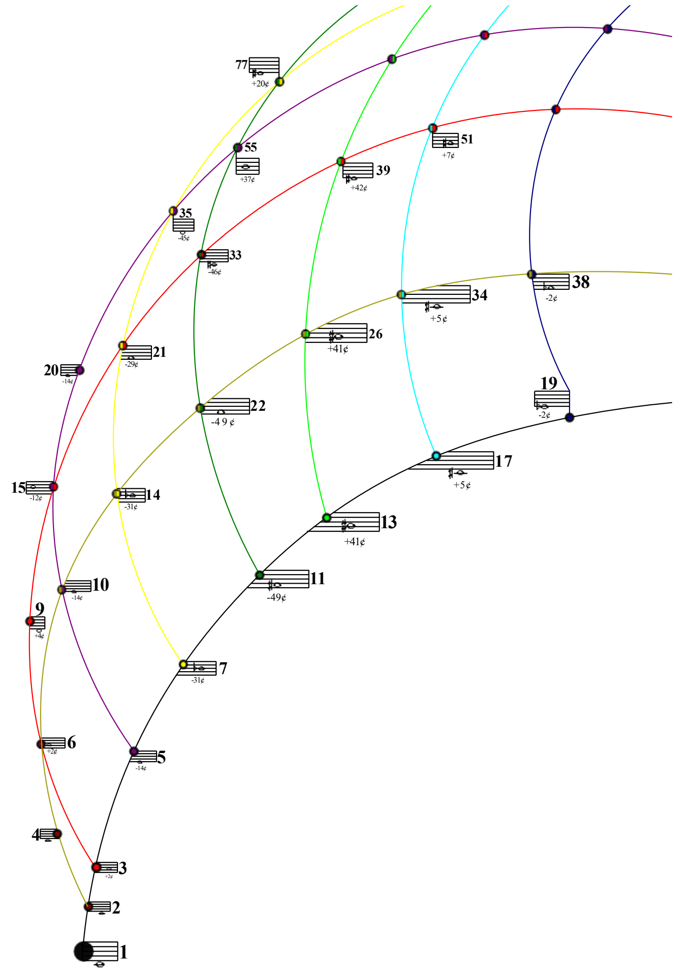

Figure 3 – The structure of harmonic series in Wilson (NEIMOG, 2021, p. 60).

Note that a new harmonic series (with new pitch classes) is born in all prime numbers. It has the same interval structure as the harmonic series over the number 1. Figure 3 has the numbers 6, 9, and 15 connected because, in the decomposition by prime numbers, they share the number 3, same for the numbers 7, 14, and 21 for sharing 7. In this perspective, we approach the harmonic conception of Johnston, where new prime numbers add new psychoacoustical meaning, and James Tenney, wherein the harmonic space, 'each dimension within the space is defined by a prime number' (Young 1988, 206).

Based on this point of view, it is possible to incorporate one approach grounded on Exarchos (2007) and apply it to the sieve result. This method can provide tools for organizing and choosing pitches, building melodic contours, symmetrical chords/timbres, changing partials of timbres with some symmetrical chords, among other procedures.

The next topic will present how we link the JI and the sieves world.

2. The sieves of Xenakis used in a JI context

One of the essential links between the Exarchos approach to sieves and the approach presented to JI is the decomposition of non-prime numbers. First, Exarchos proposes that sieves with no prime modulus can be decomposed using intersections between two or more sieves. For example, the sieve 35_0 could be constructed using $7_0 \cap 5_0$. This decomposition can "enable transformations that might not be as obvious in the actual scale" (Exarchos, 2007, p. 74). If we again see the geometric structure by Wilson, the vertice (7 5) represent the same idea in one different context. The harmonic series build in 5 (5, 10, 15, … 35) and 7 (7, 14 … 35) have one intersection (harmonic in common) in the number 35. Because of that, the number 35 can be interpreted as 7º harmonic of the number 5 or the

5º harmonic of 7. So, if we are in one harmonic series in C, the harmonic 35 (D+5¢) is at the same interval distance from 5 that Bb-31¢ (7º harmonic) is from C (fundamental) or the same distance from 7 that E-14¢ is from C. Therefore, like when we work with analysis of sieves like proposed by Exarchos, we can do "transformations that might not be as obvious in the actual scale." For example, 105 becomes not a high harmonic, but the 7º harmonic of the number 15, the 5º harmonic of 21, and 3º of 35. Thus, the pitches that can be interpreted in different ways can help the JI music not become 'grabbed' in one fundamental pitch, building a multi-dimensional space (the idea of the CPS) using these notes as pivots to modulate between harmonic series.

One possible next step is to build sieves where the harmonic result intersects one or more harmonic series in one already implicit mode in the build. For example, the sieve $((7_{35} \cap 17_{34}) \cup (19_{38} \cap 11_{33}))$ gives (77 133 154 187 231 266 308 323 374 385 399 462), with the decomposition, we will have connections between the harmonic series in 7, 17, 19, and 11, including yet 5 and 3. See the result of the OM object *prime-decomposition* that will give all these intersections between different harmonic series.

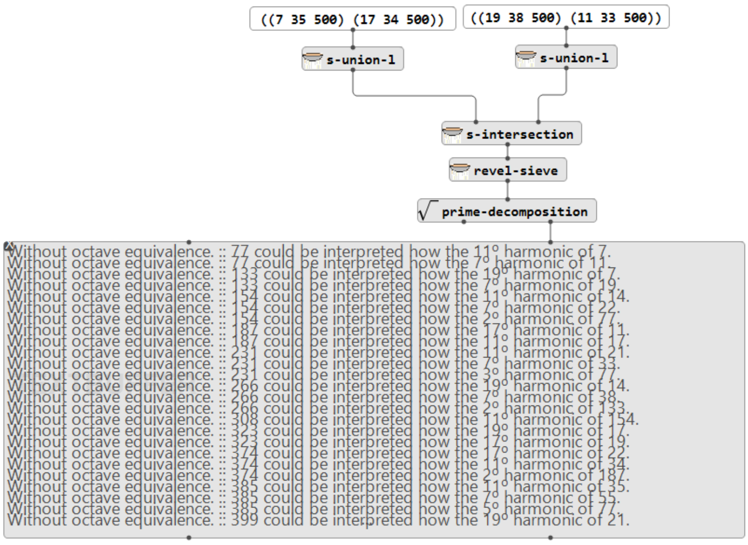

Figure 4 - Examples of the sieve analysis using the object prime-decomposition.

Three observations: First, the number starting each set needs to belong to the harmonic series of the modulus, 35 belongs to the harmonic series of 7, 34 belongs to 17, and so on in the sieve $((7_{35} \cap 17_{34}) \cup (19_{38} \cap 11_{33}))$. Second, it is not applicable in the 'orthodox' Diamond Theory because we cannot choose the numbers, just the limit. Because of that, we remove the limit concept of the Diamond Theory, allowing the choice of the numbers that will compose it. To use it in the OM environment, we need the object *diamond-identity*. We are working on a visual representation available in the next version of OM-JI. 3) It is common in the Diamond to work with pitch classes and not keep the note's octave. The octave reduction will not apply in this research context.

Once we have the sieves and the JI structures linked with some symmetry properties, we can use the sieves to build melodic contours (link of examples: https://bit.ly/3CwW0iq)

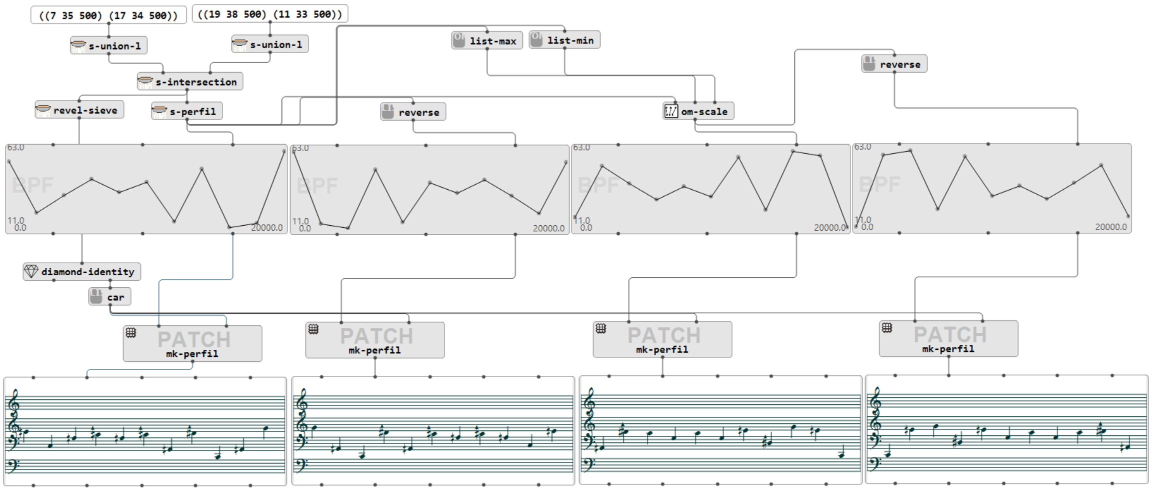

Figure 5 - Patch example to create melodic contours with sieves.

Use different sieves and Just Intonation Systems to modulate <https://bit.ly/2Y1xNOg> between then.

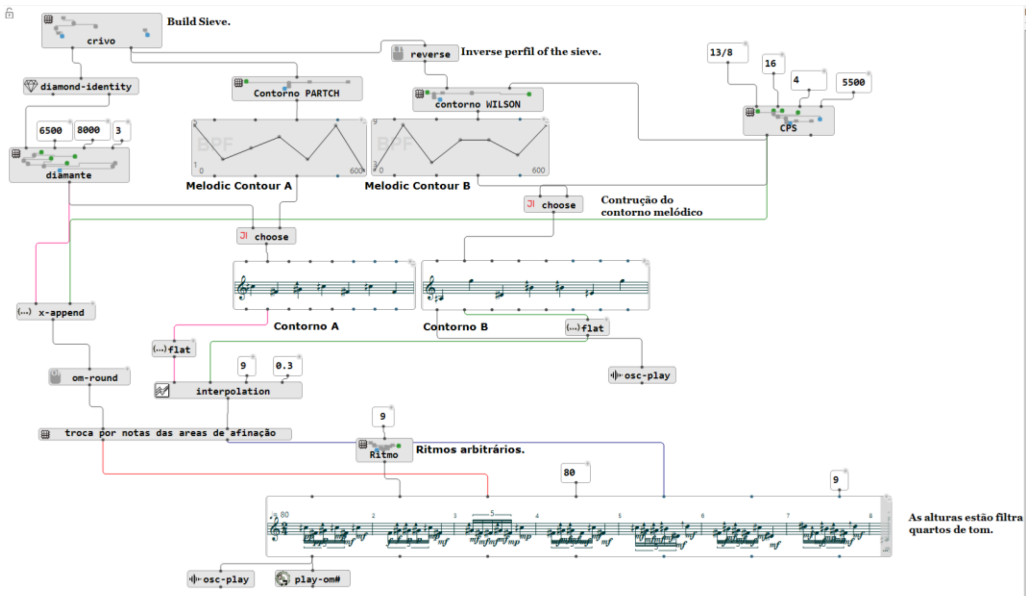

Figure 6 - Patch that creates a modulation between two different JI systems.

Another possibility is to use JI structures and the sieves in the timbre approach using the object *interval-sob* of OM-JI. It will stack some interval following the *perfil* of some sieve (https://bit.ly/368lGFL).

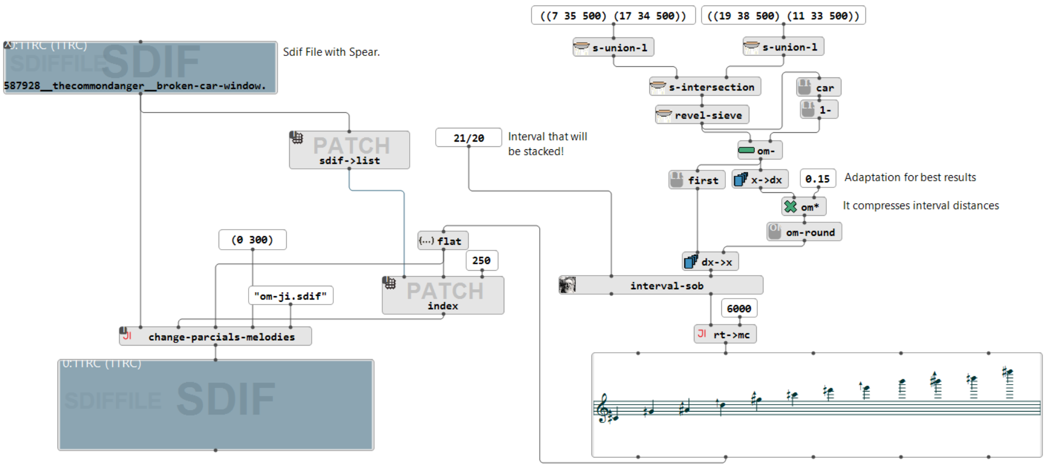

Figure 7 - This algorithm is best introduced in Neimog et al. (2022).

As shown, when we incorporate the sieves of Iannis Xenakis in the JI context, we can build ways to work with melodic contours, symmetries, timbre manipulations, and others. Mainly with the analysis method proposed by Exarchos (2007, 2011) and the connections between decomposed prime-numbers, we can organize pitches and build coherence without using the intervals set often used in JI context (3/2, 4/3, 9/8, 5/4 and others that are(was) the basis of tonal music), mainly when we work with larger prime numbers.

I believe that think Just Intonation music linking it to sieves, timbre, electroacoustic, maybe in the future some probabilities properties, could help composers of the mixed and acousmatic music take advantage of the possibilities of using any pitch by mixing tempered and JI approaches and adding, more often, the tunning thinking in electroacoustic music. Like Roads (2016, p. 237) claims: "Microtonal harmonies can become so complex that they mutate into timbre and texture. With so many pieces in free intonation, it is clear that pitch need not always be aligned to the grid of a fixed scale or intonation."

3. The compositional approach in the CAC environments

We will briefly show the two libraries developed in OpenMusic and OM-Sharp environments. The first is called OM-JI (of Just Intonation), and the second is called OM-Sieves, with the basis of Sieves (*cribles*) used in MathTools (by Carlos Agon and Moreno Andreatta) adding some features based on the work of Exarchos *et al.* (2011) and Ariza (2005).

3.1. OM-JI: Partch, Johnston e Wilson theories

The Just Intonation tools developed are divided into four parts. The first one is some of the essential and personal tools used: The objects included are:

1. *rt->mc*: convert the ratio to midicents;
2. *range-reduce* and *rt-octave*: reduce the range (midicents and ratios respectively) to some limits defined;
3. *filter-ac-inst*: Rather than use JI pitches "flattened" to quarter-tons or eight-tones notes, this object will filter pitches that can be played using fingering diagrams found in books like *Techniques of flute playing*, it can help keep the lack of beat (characteristic of JI) without the need for a group of specialized interpreters.
4. *Modulation-notes* and *modulation-notes-fund*: Based on the idea of Huey (2017), where he claims that are pitches in common between two different microtonal systems in String Quartet no. 5 of Johnston, these objects show the notes in common between two other microtonal systems. The first object without any modification of the fundamental of the pitches. In the

second object, it is possible to change the fundamentals.

5. *Ji-change-notes*: present the idea to change notes in a *sdif* file; the main idea is to retuning multiphonics. This object was described in Neimog *et al.* (2022).

In the second part, we have all the objects to build the Diamond theory of Harry Partch; the third is the objects used in Ben Johnston's theory presented in Johnston (2006), the fourth part is the objects used to build MOS and CPS theory, by Erv Wilson. The object has examples available on the website: https://www.ufjf.br/comus/cac_patches/, and all the procedures are described in detail in Neimog (2021).

3.2. OM-Sieves

The base of OM-Sieves is developed in MathTools by Carlos Agon and Moreno Andreatta. We added some objects to facilitate the interaction between composer and OM environment, and we changed all the objects of *crible* to *sieve* (French to English).

1. With the objects added, it is possible to build sieves more quickly:

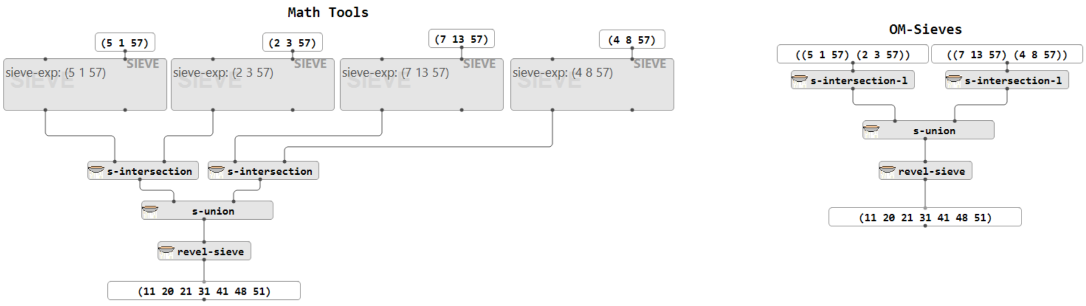

Figure 8 - Comparison between the same procedure in MathTools and OM-Sieves.

2. It is possible to use the syntax proposed by Ariza (2005). Nevertheless, in the OM environment, it is impossible to use the symbol '|', then I changed the union symbol to '*u*' and the intersection symbol to *i*. Then *24@23* | *30@3* | *104@70* | *0@0* became (24@23 u 30@3 u 104@70 u 0@0) and 3@2 & 4@7 & 6@11 & 8@7 | 6@9 & 15@18 | 13@5 & 8@6 & 4@2 | 6@9 & 15@19 became ((3@2 i 4@7 i 6@11 i 8@7) u (6@9 i 15@18) u (13@5 i 8@6 i 4@2) u (6@9 i 15@19)). Ariza does not use the parenthesis structure in the second sieve; however, it is used by Xenakis (1992, p. 274).

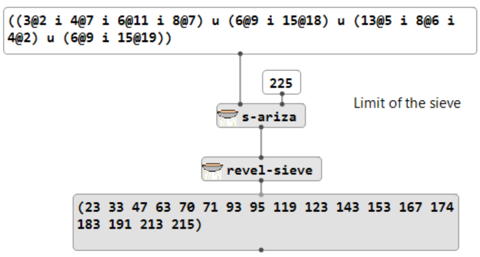

Figure 9 - Example of the creation of sieves using Ariza's (2005) syntax.

3. Also, based on Ariza (2005, p. 45), we add the possibility to discover the unions used in one sieve with the object *s-decompose*:

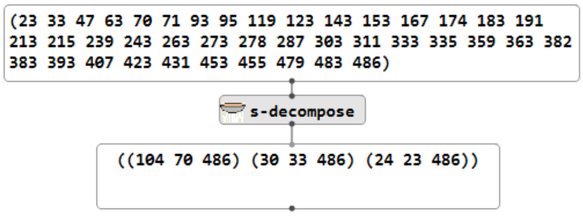

Figure 10 – The patch exemplifies the decomposition of one sieve in unions.

4. We add some of the analysis developed by Exarchos, like discovering with what limit will the sieve's *perfil* be symmetrical (palindrome):

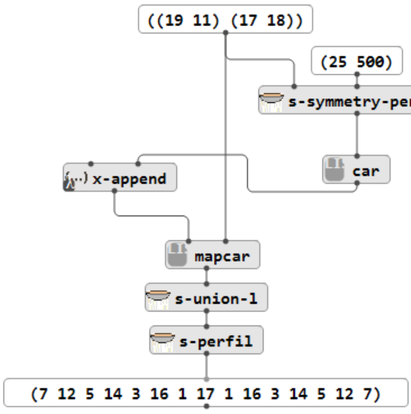

Figure 11 - It uses s-symmetry-perfil to find symmetrical sieves (Exarchos *et al.*, 2011).

5. It is possible to do the decomposition of sieves with non-prime modules.

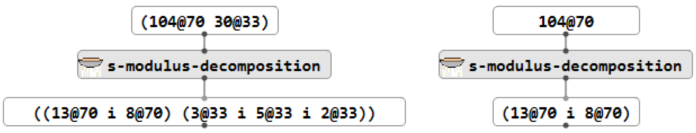

Figure 12 - The decomposition of non-prime modules proposed by Exarchos et al. (2011).

These are the main objects included in OM-Sieves. OM-JI and OM-Sieves are available to OpenMusic and OM-Sharp environment on the link https://github.com/charlesneimog/OM-JI and https://github.com/charlesneimog/OM-Sieves. If there is some doubt about the use or suggestion, you can write me.

Acknowledgements

I want to thank the Federal University of Juiz de Fora for the financial support given to this research.

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